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Fuzzy bi-quasi interior ideals of Γ-semirings

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Abstract. Generalization of ideals and fuzzy ideals of algebraic structures has shown to be both exciting and valuable to mathematics. In this paper, we introduce the notion of a fuzzy bi-quasi interior ideal as a generalization of fuzzy ideals, fuzzy bi-quasi ideals, fuzzy quasi-interior ideals, and fuzzy bi-interior ideals of a Γ-semiring. We prove that every fuzzy quasi-interior ideal is a fuzzy bi-quasi interior ideal. We also show that union and intersection of fuzzy bi-quasi interior ideals are fuzzy bi-quasi interior ideals. We characterize the regular Γ-semiring in terms of fuzzy bi-quasi interior ideals and study some of the properties. We prove that if M is a regular Γ-semiring and μ is a Γ-subsemiring, then fuzzy ideal, fuzzy quasi-interior ideal, fuzzy bi-quasi interior ideal, and fuzzy tri-ideal are equivalent.

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1. Introduction

Algebraic structures play an important role in mathematics with wide applications in many fields such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces. This provides motivation to researchers to review various concepts and results of abstract algebra in the broader framework of fuzzy setting. Semiring is an algebraic structure that commonly generalizes rings and distributive lattices, it was first introduced by Vandiver. Semiring is a universal algebra with two binary operations called addition and multiplication, where one of them is distributive over the other. A natural example of a semiring is the set of all-natural numbers under the usual addition and multiplication of numbers. The theory of rings and semigroups considerably impacted the development of the theory of semirings. In structure, semirings lie between semigroups and rings. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semirings as the basic algebraic structure used in formal languages, theoretical computer science, solutions of graph theory, optimization theory, automata, and coding theory.

The notion of a Γ-ring was introduced by Nobusawa [1] as a generalization of a ring in 1964. The notion of a Γ-semiring was introduced by Rao [2], which is a generalization of Γ-rings, rings, ternary semirings and semirings. The set of all negative integers Z is not a semiring with respect to usual addition and multiplication, but Z forms a Γ-semiring, where $\Gamma = Z$. The one and two-sided ideals are the central concepts in ring theory. We know that the notion of a one-sided ideal of any algebraic structure is a generalization of the notion of an ideal. The quasi-ideals generalize left and right ideals, whereas the bi-ideals generalize quasi-ideals. Lajos introduced the notion of bi-ideals in semigroups. Iseki introduced the quasi-ideal for a semiring.

As a further generalization of ideals, Steinfeld [3] first introduced the notion of quasi-ideals for semigroups and then for rings. We know that the notion of a bi-ideal in semirings is a special case of (m, n) ideal introduced by Lajos. Good and Hughes [4] introduced the concept of bi-ideals for a semigroup.Rao [5, 6, 7, 8, 9, 10] introduced and studied the properties of bi-quasi ideals, bi-interior ideals, quasi- interior ideals, tri-ideals, weak-interior ideals, tri-quasi ideals of Γ-semirings, Γ-semigroups, semirings, and semigroups as a generalization of ideals, bi-ideals, quasi-ideals, and interior ideals of algebraic structures.

Zadeh [11] in 1965 developed the fuzzy set theory. Many papers on fuzzy sets appeared, showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. Mandal [12] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring. Rao et al. [13, 14, 15, 16, 17, 18, 19] introduced the notions of fuzzy bi-quasi ideals, fuzzy(soft) bi-interior ideals, fuzzy(soft) tri-ideals, fuzzy(soft) quasi-interior ideals and studied their properties. This paper aims to introduce the notion of a fuzzy bi-quasi interior ideal of a Γ-semiring and characterize the regular Γ-semiring in terms of fuzzy bi-quasi interior ideals of a Γ-semiring.

2. Preliminaries

This section, recalls some fundamental concepts and definitions necessary for this paper.

Definition 2.1 ([4])**.** A set S together with two associative binary operations addition and multiplication (denoted by $+$ and $\cdot$ respectively) called a *semiring*, provided that

(i) addition is a commutative operation,
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Definition 2.2 ([22])**.** Let M and Γ be two non-empty sets. Then M is called a Γ-*semigroup*, if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (x, α, y) will be denoted by $x\alpha y$ for all $x, y \in M, \alpha \in \Gamma$) such that it satisfies

$$x\alpha(y\beta z) = (x\alpha y)\beta z \text{ for all } x, y, z \in M, \alpha, \beta \in \Gamma.$$

Definition 2.3 ([20])**.** Let $(M,+)$, $(\Gamma,+)$ be commutative semigroups and M a Γ-semigroup. Then M is called a Γ-*semiring*, if it satisfies the following conditions: for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$,

(i) $x\alpha(y+z) = x\alpha y + x\alpha z$,
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha+\beta)y = x\alpha y + x\beta y$.

Definition 2.4 ([2])**.** A Γ-semiring M is said to *have zero element*, if there exists an element $0 \in M$ such that $0+x = x = x+0$ and $0\alpha x = x\alpha 0 = 0$ for all $x \in M, \alpha \in \Gamma$.

Definition 2.5 ([21])**.** Let M be a Γ-semiring. An element $a \in M$ is said to be a *regular element* of M, if there exist $x \in M, \alpha,\beta \in \Gamma$ such that $a = a\alpha x\beta a$.

If every element of M is a regular, then M is said to be a *regular* Γ-*semiring*.

Definition 2.6 ([21])**.** Let M be a Γ-semiring. An element $a \in M$ is said to be *idempotent* of M, if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$.

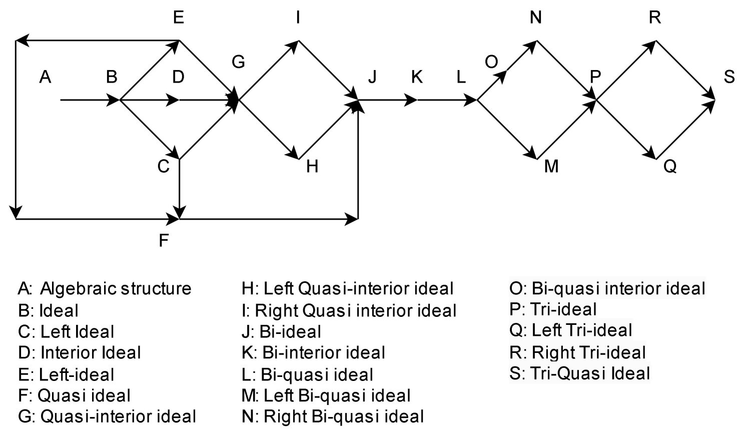
If every element of M is an idempotent of M, then M is said to be an *idempotent* Γ-*semiring*.

Definition 2.7 ([5, 6, 8, 10])**.** A non-empty subset A of Γ -semiring M is called a:

(i) *quasi-ideal* of M, if B is a Γ-subsemiring of M and $B\Gamma M \cap M\Gamma B \subseteq B$,
(ii) *bi-interior ideal* of M, if B is a Γ-subsemiring of M and $M\Gamma B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$,
(iii) *left bi-quasi ideal* (*right bi-quasi ideal*) of M, if B is a Γ-subsemiring of $(M,+)$ and $M\Gamma B \cap B\Gamma M\Gamma B \subseteq B$ ($B\Gamma M \cap B\Gamma M\Gamma B \subseteq B$),
(iv) *bi quasi ideal* of M, if B is a left bi- quasi ideal and a right bi- quasi ideal of M,
(v) *left quasi-interior ideal* (*right quasi-interior ideal*), of M if B is a Γ-subsemiring of M and $M\Gamma B\Gamma M\Gamma B \subseteq B$ ($B\Gamma M\Gamma B\Gamma M \subseteq B$),
(vi) quasi-interior ideal of M if B is a Γ-subsemiring of M and B is a left quasi-interior ideal and a right quasi-interior ideal of M.
(vii) *bi-quasi interior ideal* of M, if B is a Γ-subsemiring of M and $B\Gamma M\Gamma B\Gamma M\Gamma B \subseteq B$,
(viii) *left tri-ideal* (*right tri-ideal*) of M, if B is a Γ-subsemiring of M and $B\Gamma M\Gamma B\Gamma B \subseteq B$ ($B\Gamma B\Gamma M\Gamma B \subseteq B$),
(ix) *tri-ideal* of M if B is a Γ-subsemiring of M, and $B\Gamma M\Gamma B\Gamma B \subseteq B$ and $B\Gamma B\Gamma M\Gamma B \subseteq B$,
(x) *tri-quasi ideal* of M, if B is a Γ-subsemiring of M and $B\Gamma B\Gamma M\Gamma B\Gamma B \subseteq B$,
(xi) $left(right)$ *weak-interior ideal* of M, if B is a $\Gamma-$subsemiring of M and $M\Gamma B\Gamma B \subseteq B (B\Gamma B\Gamma M \subseteq B)$,

Figure(1): The inter relationships between some generalization of ideal mentioned before are visualized in figure(1). (Arrows indicates proper inclusions. That is if X and Y are ideals then $X \to Y$ means $X \subset Y$.)

Theorem 2.8 ([20])**.** *Let M be Γ-semiring. Then M is a regular Γ-semiring if and only if $A\Gamma B = A \cap B$, for any right ideal A and left ideal B of M.*



Definition 2.9 ([11])**.** Let M be a non-empty set. Then a mapping $\mu : M \to [0,1]$ is called a *fuzzy subset* of M. If μ is not a constant function, then μ is called a *non-empty fuzzy subset* of M. For two fuzzy sets f and g in M, $f \cup g$ and $f \cap g$ are defined as follows: for each $z \in M$,

$$f \cup g(z) = max\{f(z), g(z)\},\ f \cap g(z) = min\{f(z), g(z)\}.$$

Definition 2.10 ([14])**.** Let μ be a fuzzy subset of a non-empty set M. Then for $t \in [0,1]$, the set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called a *level subset* of M with respect to t.

Definition 2.11 ([14])**.** For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$, for all $x \in M$.

Definition 2.12 ([14])**.** Let f and g be fuzzy subsets of a $\Gamma-$semiring M. Then $f \circ g$ and $f + g$ are defined by: for each $z \in M$,

$$f \circ g(z) = \begin{cases} \underset{z=x\alpha y,\ x,y\in M,\alpha\in\Gamma}{sup} \{min\{f(x), g(y)\}\} & \\ 0 & \text{otherwise,} \end{cases}$$

$$f + g(z) = \begin{cases} \underset{z=x+y,\ x,y\in M,\alpha\in\Gamma}{sup} \{min\{f(x), g(y)\}\} & \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.13 ([13])**.** Let A be a non-empty subset of M. The *characteristic function* of A is a fuzzy subset of M, defined by: for all $x \in X$,

$$\chi_{_A}(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Definition 2.14 ([17, 18, 19, 21])**.** Let M be a Γ-semiring. A fuzzy subset μ of M is called a:

(i) *fuzzy quasi-ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}, \mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$,

(b) $\mu \circ \chi_M \cap \chi_M \circ \mu \subseteq \mu$,

(ii) *fuzzy left* (*right*) *bi-quasi ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}, \mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$,

(b) $\chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu\big(\mu \circ \chi_M \circ \mu \circ \chi_M \subseteq \mu\big)$,

(iii) *fuzzy tri-quasi ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$,

(b) $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$,

(iv) *fuzzy bi-interior ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$,

(b) $\chi_M \circ \mu \circ \chi_M \cap \mu \circ \chi_M \circ \mu \subseteq \mu$,

(v) *fuzzy tri-ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$,

(b) $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$,

(vi) *fuzzy quasi-interior ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$,

(b) $\chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$,

(vii) *fuzzy weak-interior ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(a) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$,

(b)$\mu \circ \mu \circ \chi_M \subseteq \mu$.

3. Fuzzy bi-quasi interior ideals of Γ-semirings

In this section, we introduce the notion of fuzzy bi-quasi interior ideals as a generalization of fuzzy ideals, fuzzy bi-interior ideals, fuzzy quasi-ideals, fuzzy bi-quasi ideals of a Γ-semiring and study the properties of fuzzy bi-quasi interior ideals.

Definition 3.1. A fuzzy subset μ of a Γ-semiring M is called a *fuzzy bi-quasi-interior ideal*, if

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in M, \alpha \in \Gamma$,
(iii) $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$.

Example 3.2. Let $M = \{a, b, c, d, e\}, \Gamma = \{\alpha, \beta\}$. Define the binary operations "$+$" on M and "$\cdot$" with the following Cayley tables:

$+$	a	b	c	d	e
a	a	b	c	d	e
b	b	a	a	a	a
c	c	a	a	a	a
d	d	a	a	a	a
e	e	a	a	a	a

$+$	α	β
α	α	α
β	α	β

Then $(M, +)$ and $(\Gamma, +)$ are commutative semigroups.

Define the ternary operation $(x\ \alpha\ y) \rightarrow x\alpha y$ $(M \times \Gamma \times M \rightarrow M)$ is defined by:

α	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	d	d
c	a	d	c	d	e
d	a	d	a	d	d
e	a	d	c	d	e

β	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	d	d
c	a	d	c	d	e
d	a	d	a	d	d
e	a	d	c	d	e

Then M is a Γ-semiring. Let $B = \{a, c\}$. Then B is a bi-quasi interior ideal of M.

(1) Consider the fuzzy subset μ of M defined as follows:

$$\mu(a) = 1, \mu(b) = 0.8, \mu(c) = 0.3, \mu(d) = 0.5, \mu(e) = 0.7.$$

Then μ is a fuzzy bi-quasi interior ideal of M.

(2) Now consider the fuzzy subset of M given by:

$$\mu(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{if } x \notin B. \end{cases}$$

Then μ is a fuzzy bi-quasi interior ideal of M.

Theorem 3.3. *Let M be a Γ-semiring and μ be a nonempty fuzzy subset of M. Every fuzzy right ideal of M is a fuzzy right-quasi interior ideal of M.*

Proof. Let μ be a fuzzy right ideal of M and $x \in M$. Then we have

$$\begin{aligned}
\mu \circ \chi_M(x) &= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \{\min\{\mu(a), \chi_M(b)\}\} \\
&= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \{\mu(a)\} \\
&\leq \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \{\mu(a\alpha b)\} \\
&= \mu(x).
\end{aligned}$$

Thus $\mu \circ \chi_M(x) \leq \mu(x)$. On the other hand, we have

$$\begin{aligned}
\mu \circ \chi_M \circ \mu \circ \chi_M(x) &= \sup_{x=u\alpha v,\ u,v\in M,\alpha\in\Gamma} \{\min\{\mu \circ \chi_M(u), \mu \circ \chi_M(v)\}\} \\
&\leq \sup_{x=u\alpha v,\ u,v\in M,\alpha\in\Gamma} \{\min\{\mu(u), \mu(v)\}\} \\
&= \mu \circ \mu(x) \\
&\leq \mu(x).
\end{aligned}$$

So μ is a fuzzy right-quasi interior ideal of M. $\square$

Corollary 3.4. *Let M be a Γ-semiring, and μ be a nonempty fuzzy subset of M. Every fuzzy (left) ideal of M is a fuzzy (left)-quasi interior ideal of M.*

Theorem 3.5. *Let M be a Γ-semiring, and μ a nonempty fuzzy subset of M. If μ is a fuzzy left-ideal of M, then μ is a fuzzy bi-ideal of M.*

Proof. Suppose μ is a fuzzy left-ideal of M. Let $z \in M$. Then we have

$$\begin{aligned} \chi_M \circ \mu(z) &= \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\min\{\chi_M(l), \mu(m)\}\} \\ &= \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\min\{1, \mu(m)\}\} \\ &= \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\mu(m)\} \\ &\leq \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\mu(lm)\} \\ &= \mu(z), \end{aligned}$$

$$\begin{aligned} \mu \circ \chi_M \circ \mu(z) &= \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\min\{\mu(l), \chi_M \circ \mu(m)\}\} \\ &\leq \sup_{z=l\alpha m,\ l,m\in M,\alpha\in\Gamma} \{\min\{\mu(l), \mu(m)\}\} \\ &= \mu(z). \end{aligned}$$

Thus $\mu \circ \chi_M \circ \mu(z) \leq \mu(z)$. So $\mu \circ \chi_M \circ \mu \subseteq \mu$. Hence μ is a fuzzy bi-ideal of M. □

Theorem 3.6. *Let M be a Γ-semiring, and μ a nonempty fuzzy subset of M. Every fuzzy right quasi-interior ideal of M is a fuzzy bi-quasi interior ideal of M.*

Proof. Let μ be a fuzzy right quasi-interior ideal of M and $x \in M$. Then we get

$$\mu \circ \chi_M \circ \mu \circ \chi_M \subseteq \mu.$$

On the other hand, we have

$$\begin{aligned} \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu(x) &= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \{\min\{\mu \circ \chi_M \circ \mu \circ \chi_M(a), \mu(b)\}\} \\ &\leq \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \{\min\{\mu(a), \mu(b)\}\} \\ &= \mu \circ \mu(x) \\ &\leq \mu(x). \end{aligned}$$

Thus μ is a fuzzy bi-quasi interior ideal of M. □

Corollary 3.7. *Every fuzzy (left) quasi-interior ideal of a Γ-semiring M is a fuzzy bi-quasi-interior ideal of M.*

Theorem 3.8. *Let M be a Γ-semiring and μ be a non-empty fuzzy subset of M. Then μ is a fuzzy bi-quasi interior ideal of M if and only if the level subset μ_t of μ is a bi-quasi interior ideal of M for every $t \in [0,1]$, where $\mu_t \neq \phi$.*

Proof. Suppose μ is a fuzzy bi-quasi interior ideal of M and $\mu_t \neq \phi$ for each $t \in [0,1]$. Let $a, b \in \mu_t$. Then $\mu(a) \geq t,\ \mu(b) \geq t$. Thus for each $\alpha \in \Gamma$,

$$\mu(a+b) \geq \min\{\mu(a), \mu(b)\} \geq t,\ \mu(a\alpha b) \geq \min\{\mu(a), \mu(b)\} \geq t.$$

So $a + b \in \mu_t,\ a\alpha b \in \mu_t$. Hence μ_t is a Γ-subsemiring of M.

Now let $x \in \mu_t\Gamma M\Gamma\mu_t\Gamma M\Gamma\mu_t$. Then $x = a\alpha b\beta c\eta d\theta e$, where $c \in M, a, b, d \in \mu_t, \alpha, \beta, \eta, \theta \in \Gamma$. Thus $\mu_t\Gamma M\Gamma\mu_t\Gamma M\Gamma\mu_t(x) \geq t$. So $x \in \mu_t$. Hence we have

$$\mu_t\Gamma M\Gamma\mu_t\Gamma M\Gamma\mu_t \subseteq \mu_t.$$

Therefore μ_t is a bi-quasi interior ideal of M.

Conversely, suppose that μ_t is a bi-quasi interior ideal of M for all $t \in Im(\mu)$. Let $x, y \in M, \alpha \in \Gamma, \mu(x) = t_1, \mu(y) = t_2$ and $t_1 \geq t_2$. Then $x, y \in \mu_{t_2}$. Thus $x + y \in \mu_{t_2},\ x\alpha y \in \mu_{t_2}$. So we have

$$\mu(x+y) \geq t_2 = \min\{t_1, t_2\} = \min\{\mu(x), \mu(y)\},\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}.$$

Hence we have $\mu_l \Gamma M \Gamma \mu_l \Gamma M \Gamma \mu_l \subseteq \mu_l$ for all $l \in Im(\mu)$.

Now let $t = \min\{Im(\mu)\}$. Then $\mu_t \Gamma M \Gamma \mu_t \Gamma M \Gamma \mu_t \subseteq \mu_t$. Thus $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$. So μ is a fuzzy bi-quasi interior ideal of M. □

Theorem 3.9. *Let M be a Γ-semiring. If μ is a fuzzy bi-ideal of M, then μ is a fuzzy bi-quasi interior ideal of M.*

Proof. Suppose μ is a fuzzy bi-ideal of M. Then $\mu \circ \chi_M \circ \mu \subseteq \mu$. Let $z \in M$. Then we have

$$\begin{aligned} \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu(z) &= \sup_{z = l\alpha m,\ l,m \in M, \alpha \in \Gamma} \{\min\{\mu \circ \chi_M \circ \mu(l), \chi_M \circ \mu(m)\}\} \\ &\leq \sup_{z = l\alpha m,\ l,m \in M, \alpha \in \Gamma} \{\min\{\mu(l), \chi_M \circ \mu(m)\}\} \\ &= \mu \circ \chi_M \circ \mu(z) \\ &\leq \mu(z). \end{aligned}$$

Thus μ is a fuzzy bi-quasi interior of M. □

Theorem 3.10. *Let I be a non-empty subset of a Γ-semiring M and χ_I the characteristic function of I. Then I is a bi-quasi interior ideal of M if and only if χ_I is a fuzzy bi-quasi interior ideal of M.*

Proof. Suppose I is a bi-quasi interior ideal of M. Obviously, χ_I is a fuzzy Γ-subsemiring of M. Then $I\Gamma M \Gamma I \Gamma M \Gamma I \subseteq I$. Thus

$$\begin{aligned} \chi_I \circ \chi_M \circ \chi_I \circ \chi_M \circ \chi_I &= \chi_{I\Gamma M\Gamma I\Gamma M\Gamma I} \\ &\subseteq \chi_I. \end{aligned}$$

So χ_I is a fuzzy bi-quasi interior ideal of M.

Conversely, suppose χ_I is a fuzzy bi-quasi interior ideal of M. Then I is a Γ-subsemiring of M. Thus we have $\chi_I \circ \chi_M \circ \chi_I \circ \chi_M \circ \chi_I \subseteq \chi_I$. So $\chi_{I\Gamma M\Gamma I\Gamma M\Gamma I} \subseteq \chi_I$. Hence $I\Gamma M \Gamma I \Gamma M \Gamma I \subseteq I$. Therefore I is a bi-quasi interior ideal of M. □

Theorem 3.11. *If μ and λ are fuzzy bi-quasi interior ideals of a Γ-semiring M, then $\mu \cup \lambda$ is a fuzzy bi-quasi interior ideal of M.*

Proof. Suppose μ and $\lambda$are fuzzy bi-quasi interior ideals of M. Let $x, y \in M, \alpha \in \Gamma$. Then we have

$$\begin{aligned}
\mu \cup \lambda(x+y) &= \max\{\mu(x+y), \lambda(x+y)\} \\
&\geq \max\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\
&= \min\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\} \\
&= \min\{\mu \cup \lambda(x), \mu \cup \lambda(y)\}, \\
\mu \cup \lambda(x\alpha y) &= \max\{\mu(x\alpha y), \lambda(x\alpha y)\} \\
&\geq \max\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\
&= \min\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\} \\
&= \min\{\mu \cup \lambda(x), \mu \cup \lambda(y)\}.
\end{aligned}$$

Then $\mu \cup \lambda$ is a fuzzy Γ-subsemiring. Moreover, for each $x \in M$, we get

$$\begin{aligned}
\chi_M \circ \mu \cup \lambda(x) &= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \min\{\chi_M(a), \mu \cup \lambda(b)\} \\
&= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \min\{\chi_M(a), \max\{\mu(b), \lambda(b)\}\} \\
&= \sup_{x=a\alpha b,\ a,b\in M,\alpha\in\Gamma} \max\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\} \\
&= \max\{\sup_{x=a\alpha b} \min\{\chi_M(a), \mu(b)\}, \sup_{x=a\alpha b} \min\{\chi_M(a), \lambda(b)\}\} \\
&= \max\{\chi_M \circ \mu(x), \chi_M \circ \lambda(x)\} \\
&= \chi_M \circ \mu \cup \chi_M \circ \lambda(x).
\end{aligned}$$

Thus $\chi_M \circ \mu \cup \chi_M \circ \lambda = \chi_M \circ (\mu \cup \lambda)$. On the other hand, we have

$$\begin{aligned}
&\mu \cup \lambda \circ \chi_M \circ \mu \cup \lambda(x) \\
&= \sup_{x=a\alpha b,,\ a,b\in M,\alpha\in\Gamma} \min\{\mu \cup \lambda(a), \chi_M \circ \mu \cup \lambda(b)\} \\
&= \sup_{x=a\alpha b,,\ a,b\in M,\alpha\in\Gamma} \min\{\max\{\mu(a), \lambda(a)\}, \quad \max\{\chi_M \circ \mu(b), \chi_M \circ \lambda(b)\}\}
\end{aligned}$$

$$\begin{aligned}
&= \sup_{x=a\alpha b,,\ a,b\in M,\alpha\in\Gamma} \min\{\max\{\mu(a), \chi_M \circ \mu(b)\}, \quad \max\{\lambda(a), \chi_M \circ \lambda(b)\}\} \\
&= \max\{\sup_{x=a\alpha b} \min\{\mu(a), \chi_M \circ \mu(b)\}, \sup_{x=a\alpha b} \min\{\lambda(a), \chi_M \circ \lambda(b)\}\} \\
&= \max\{\mu \circ \chi_M \circ \mu(x), \lambda \circ \chi_M \circ \lambda(x)\} \\
&= \mu \circ \chi_M \circ \mu \cup \lambda \circ \chi_M \circ \lambda(x),
\end{aligned}$$

$$\begin{aligned}
&\mu\cup\lambda\circ\chi_M\circ\mu\cup\lambda\circ\chi_M\circ\mu\cup\lambda(x)\\
&=\sup_{x=a\alpha b}\min\{\mu\cup\lambda\circ\chi_M\circ\mu\cup\lambda(a),\chi_M\circ\mu\cup\lambda(b)\}\\
&=\sup_{x=a\alpha b}\min\{\mu\circ\chi_M\circ\mu\cup\lambda\circ\chi_M\circ\lambda(a),\chi_M\circ\mu\cup\chi_M\circ\lambda(b)\}\\
&=\sup_{x=a\alpha b}\max\{\min\{\mu\circ\chi_M\circ\mu(a),\lambda\circ\chi_M\circ\lambda(a)\},\min\{\chi_M\circ\mu(b),\chi_M\circ\lambda(b)\}\}\\
&=\max\{\sup_{x=a\alpha b}\min\{\mu\circ\chi_M\circ\mu(a),\chi_M\circ\mu(b)\},\sup_{x=a\alpha b}\min\{\lambda\circ\chi_M\circ\lambda(a),\chi_M\circ\lambda(b)\}\}\\
&=\max\{\mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu(z),\lambda\circ\chi_M\circ\lambda\circ\chi_M\circ\lambda(z)\}\\
&=\mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu\cup\lambda\circ\chi_M\circ\lambda\circ\chi_M\circ\lambda(z).
\end{aligned}$$

So we have

$$\begin{aligned}
\mu\cup\lambda\circ\chi_M\circ\mu\cup\lambda\circ\chi_M\circ\mu\cup\lambda &= \mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu\cup\lambda\circ\chi_M\circ\lambda\circ\chi_M\circ\lambda\\
&\subseteq \mu\cup\lambda.
\end{aligned}$$

Hence $\mu\cup\lambda$ is a fuzzy bi-quasi interior ideal of M. □

Theorem 3.12. *If μ and λ are fuzzy bi-quasi interior ideals of a Γ-semiring M, then $\mu\cap\lambda$ is a fuzzy bi-quasi interior ideal of M.*

Proof. Suppose μ and λ are fuzzy bi-quasi interior ideals of M and let $x, y \in M$ and $\alpha\in\Gamma$. Then we have

$$\begin{aligned}
\mu\cap\lambda(x+y) &= \min\{\mu(x+y),\lambda(x+y)\}\\
&\geq \min\{\min\{\mu(x),\mu(y)\},\min\{\lambda(x),\lambda(y)\}\}\\
&= \min\{\min\{\mu(x),\lambda(x)\},\min\{\mu(y),\lambda(y)\}\}\\
&= \min\{\mu\cap\lambda(x),\mu\cap\lambda(y)\},
\end{aligned}$$

$$\begin{aligned}
\chi_M\circ\mu\cap\lambda(x) &= \sup_{x=a\alpha b,\ a,b\in M}\min\{\chi_M(a),\mu\cap\lambda(b)\}\\
&= \sup_{x=a\alpha b,\ a,b\in M}\min\{\chi_M(a),\min\{\mu(b),\lambda(b)\}\}\\
&= \sup_{x=a\alpha b,\ a,b\in M}\min\{\min\{\chi_M(a),\mu(b)\},\min\{\chi_M(a),\lambda(b)\}\}\\
&= \min\{\sup_{x=a\alpha b}\min\{\chi_M(a),\mu(b)\},\sup_{x=a\alpha b}\min\{\chi_M(a),\lambda(b)\}\}\\
&= \min\{\chi_M\circ\mu(x),\chi_M\circ\lambda(x)\}\\
&= \chi_M\circ\mu\cap\chi_M\circ\lambda(x).
\end{aligned}$$

Thus $\chi_M \circ \mu \cap \chi_M \circ \lambda = \chi_M \circ (\mu \cap \lambda)$. On the other hand, we get

$$\begin{aligned}
&\mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda(x) \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\mu \cap \lambda(a), \chi_M \circ \mu \cap \lambda(b)\} \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\mu \cap \lambda(a), \chi_M \circ \mu \cap \chi_M \circ \lambda(b)\} \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\min\{\mu(a), \lambda(a)\}, \min\{\chi_M \circ \mu(b), \chi_M \circ \lambda(b)\}\} \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\min\{\mu(a), \chi_M \circ \mu(b)\}, \min\{\lambda(a), \chi_M \circ \lambda(b)\}\} \\
&= \min\{\sup_{x=a\alpha b} \min\{\mu(a), \chi_M \circ \mu(b)\}, \sup_{x=a\alpha b} \min\{\lambda(a), \chi_M \circ \mu(b)\}\} \\
&= \min\{\mu \circ \chi_M \circ \mu(x), \lambda \circ \chi_M \circ \lambda(x)\} \\
&= \mu \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \lambda(x).
\end{aligned}$$

$$\begin{aligned}
&\mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda(x) = \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda(a), \chi_M \circ \mu \cap \lambda(b)\} \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\mu \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \lambda(a), \chi_M \circ \mu \cap \chi_M \circ \lambda(b)\} \\
&= \sup_{x=a\alpha b,\ a,b\in M} \min\{\min\{\mu \circ \chi_M \circ \mu(a), \lambda \circ \chi_M \circ \lambda(a)\}, \min\{\chi_M \circ \mu(b), \chi_M \circ \lambda(b)\}\} \\
&= \min\{\sup_{x=a\alpha b} \min\{\mu \circ \chi_M \circ \mu(a), \chi_M \circ \mu(b)\}, \sup_{x=a\alpha b} \min\{\lambda \circ \chi_M \circ \lambda(a), \chi_M \circ \lambda(b)\}\} \\
&= \min\{\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu(z), \lambda \circ \chi_M \circ \lambda \circ \chi_M \circ \lambda(z)\} \\
&= \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \lambda \circ \chi_M \circ \lambda(z).
\end{aligned}$$

So we have $\mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda(x) = \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \cap \lambda \circ \chi_M \circ \lambda \circ \chi_M \circ \lambda \subseteq \mu \cap \lambda$.
Hence $\mu \cap \lambda$ is a fuzzy bi-quasi interior ideal of M. □

4. Fuzzy bi-quasi interior ideals of regular Γ− semirings

Theorem 4.1. *Let M be a regular Γ-semiring. If μ is a fuzzy bi-quasi interior ideal of M, then $\mu \circ \mu = \chi_M \circ \mu = \mu \circ \chi_M = \mu$.*

Proof. Suppose μ is a fuzzy bi-quasi interior ideal of M and let $z \in M$. Since M is a regular Γ-semiring, there exist $\alpha, \beta \in \Gamma, x \in M$ such that $z = z\alpha x\beta z$. Then we have

$$\begin{aligned}
\mu \circ \mu(z) &= \sup_{z=z\alpha x\beta z} \{\{\min\{\mu(z\alpha x), \mu(z)\}\} \\
&= \sup_{z=z\alpha z} \{\mu(z)\} \\
&= \mu(z).
\end{aligned}$$

Thus $\mu \circ \mu = \mu$. On the other hand, we have

$$\begin{aligned}\chi_M \circ \mu(z) &= \sup_{z=z\alpha x\beta z}\{\min\{\chi_M(z\alpha x), \mu(z)\}\}\\ &= \sup_{z=z\alpha x\beta z}\{\min\{1, \mu(z)\}\}\\ &= \sup_{z=z\alpha x\beta z}\mu(z)\\ &= \mu(z).\end{aligned}$$

Similarly, $\mu \circ \chi_M = \mu$. So the result holds. □

Theorem 4.2. *Let M be a regular Γ-semiring and μ a nonempty fuzzy subset of M. Then μ a fuzzy bi-quasi-interior ideal of M if and only if $\mu = \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu$ for any fuzzy bi-quasi interior ideal μ of M.*

Proof. Suppose μ is a fuzzy bi-quasi-interior ideal of M. Then $\mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu \subseteq \mu$. Thus we have: for each $x \in M$,

$$\begin{aligned}\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu(x) &= \sup_{x=x\alpha y\beta x,\ y\in M,\alpha,\beta\in\Gamma}\{\min\{\mu \circ \chi_M \circ \mu(x), \chi_M \circ \mu(y\beta x)\}\}\\ &\geq \sup_{x=x\alpha y\beta x,\ y\in M,\alpha,\beta\in\Gamma}\{\min\{\mu(x), \mu(x)\}\}\\ &= \mu(x).\end{aligned}$$

Thus $\mu \subseteq \mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu$. So $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu = \mu$.

Conversely, suppose $\mu = \mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu$ for any fuzzy bi-quasi interior ideal μ of M. Let B be a bi-quasi interior ideal of M. Then χ_B be a fuzzy bi-quasi interior ideal of M. Thus we have

$$\chi_B = \chi_B \circ \chi_M \circ \chi_B \circ \chi_M \circ \chi_B = \chi_{B\Gamma M\Gamma B\Gamma M\Gamma B}.$$

So $B = B\Gamma M\Gamma B\Gamma M\Gamma B$.

Now suppose $B\Gamma M\Gamma B\Gamma M\Gamma B = B$ for all bi-quasi-interior ideals B of M and let $B = R \cap L$, where R is a right ideal and L is a left ideal of M. Then B is a bi-quasi interior ideal of M. Thus $(R \cap L)\Gamma M\Gamma(R \cap L)\Gamma M\Gamma(R \cap L) = R \cap L$. On the other hand, we have

$$\begin{aligned}R \cap L &= (R \cap L)\Gamma M\Gamma(R \cap L)\Gamma M\Gamma(R \cap L)\\ &\subseteq R\Gamma M\Gamma L\Gamma M\Gamma L\\ &\subseteq R\Gamma L\\ &\subseteq R \cap L(\text{Since } R\Gamma L \subseteq L \ \ \textit{and} \ \ R\Gamma L \subseteq R).\end{aligned}$$

So $R \cap L = R\Gamma L$. Hence M is a regular Γ-semiring. □

Theorem 4.3. *Let M be a regular Γ-semiring. If μ is a fuzzy bi-quasi interior ideal of M, then μ is a fuzzy right tri-ideal of M.*

Proof. Suppose μ is a fuzzy bi-quasi interior ideal of M. Then $\mu\circ\chi_M\circ\mu\circ\chi_M\circ\mu \subseteq \mu$. Thus By Theorem 4.1, we have

$$\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu = \mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu.$$

So μ is a fuzzy right tri-ideal of M. □

Corollary 4.4. *Let M be a regular Γ-semiring. If μ is a fuzzy bi-quasi interior ideal of M, then μ is a fuzzy (left) tri-ideal of M.*

Theorem 4.5. *Let M be a regular Γ-semiring, and μ a fuzzy subset of M. If μ is a fuzzy left tri-ideal of M, then μ is a fuzzy left ideal of M.*

Proof. Suppose μ is a fuzzy left tri-ideal of M. Then $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. Let $z \in M$. Then there exist $x \in M, \alpha, \beta \in \Gamma$ such that $z = z\alpha x\beta z$. By Theorem 4.1, we have $\mu \circ \chi_M \circ \mu \subseteq \mu$. Thus $\chi_M \circ \mu \circ \mu \subseteq \mu$. So $\chi_M \circ \mu \subseteq \mu$. Hence μ is a fuzzy left-ideal of M. □

Corollary 4.6. *Let M be a regular Γ-semiring. Then μ is a fuzzy(right) tri-ideal if and only if μ is a fuzzy (right) ideal of M.*

Theorem 4.7. *Let M be a regular Γ-semiring. If μ is a fuzzy right tri-ideal of M, then μ is a fuzzy left quasi-interior ideal of M.*

Proof. Suppose μ is a fuzzy right tri-ideal of a regular $\Gamma-$semiring M. Then $\mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu$. Thus by Theorem 4.1, we have

$$\chi_M \circ \mu \circ \chi_M \circ \mu = \mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu.$$

So μ is a fuzzy left quasi-interior ideal of M. □

Corollary 4.8. *Let M be a regular Γ-semiring. If μ is a fuzzy (right)tri-ideal of M, then μ is a fuzzy (right) quasi-interior ideal of M.*

Theorem 4.9. *Let M be a regular Γ-semiring. If μ is a fuzzy bi-quasi interior ideal of M, then μ is a fuzzy tri-quasi ideal of M.*

Proof. Suppose μ is a fuzzy bi-quasi interior ideal of M. Then $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$. Let $z \in M$. Then there exist $\alpha, \beta \in \Gamma, x \in M$ such that $z = z\alpha x\beta z$. Then $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu(z) \leq \mu(z)$. Thus by Theorem 4.1, $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(z) \leq \mu(z)$. So μ is a fuzzy tri-quasi ideal of M. □

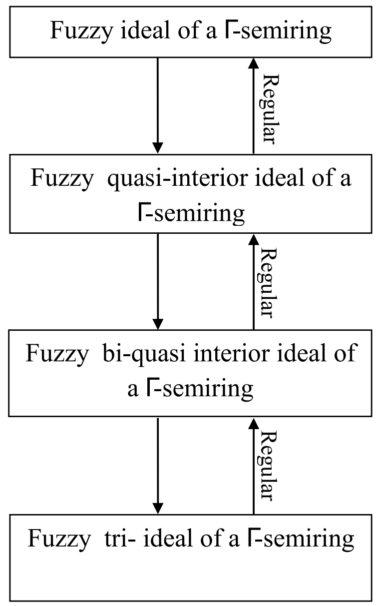
Theorem 4.10. *Let M be a regular Γ-semiring. If μ is a fuzzy Γ-subsemiring of M, then the following are equivalent:*
(1) *μ is a fuzzy ideal of M,*
(2) *μ is a fuzzy quasi-interior ideal of M,*
(3) *μ is a fuzzy bi-quasi interior ideal of M,*
(4) *μ is a fuzzy tri-ideal of M.*

Proof. (1) ⇒ (2) The proof folllows from Corollary 3.4.
(2) ⇒ (3) The proof folllows from Corollary 3.7.
(3) ⇒ (4) The proof folllows from Corollary 4.4.
(4) ⇒ (1) The proof folllows from Corollary 4.6. □

5. Conclusion

In this paper, we studied the notion of fuzzy bi-quasi interior ideals of a Γ-semiring as a generalization of fuzzy ideals, fuzzy bi-ideals, fuzzy quasi-ideals, fuzzy bi-quasi ideals, fuzzy quasi-interior ideals and fuzzy interior-ideals. We proved a fuzzy bi-ideal

Relation between these fuzzy generalization of ideals are illustrated by the following diagram where **A** $\longrightarrow$ **B** means that **A** is **B** but **B** may not be **A**.



of a Γ-semiring is a fuzzy bi-quasi interior ideal and a fuzzy tri-ideal of a regular Γ-semiring is a fuzzy left ideal and a fuzzy bi-quasi interior ideal of a regular Γ-semiring is a fuzzy tri-ideal of a Γ-semiring. We characterized the regular Γ-semiring in terms of fuzzy bi-quasi interior ideals of Γ-semirings and studied some of the properties. As a further extension we wish to study fuzzy(soft) bi-quasi interior ideals of algebraic structures.

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